Maximally smooth splines on triangulations

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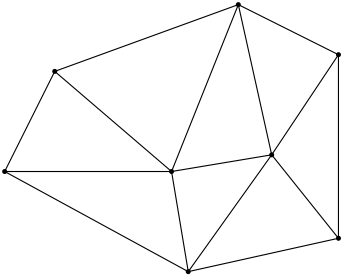
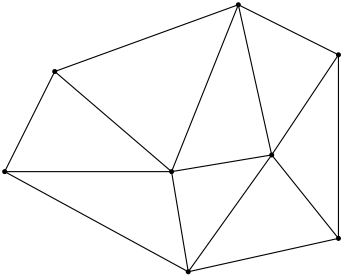
Joint work with T. Lyche and C. Manni

Goal



- Build smooth splines on arbitrary (possibly refined) triangulations
- Equip them with a good local basis
- Have efficient algorithms for their manipulation

Splines on triangulations

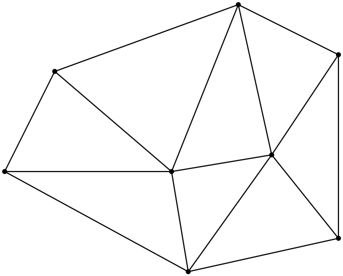


- Let $\mathcal{T}_0$ be a triangulation of a polygonal domain $\Omega \in \mathbb{R}^2$
- A spline space on $\mathcal{T}_0$:

$$\mathbb{S}_p^r(\mathcal{T}_0) := \{f \in C^r(\Omega) : f_{|\Delta} \in \mathbb{P}_p, \ \forall \ \Delta \in \mathcal{T}_0\}$$

with $\mathbb{P}_p :=$ space of polynomials of degree $\leq p$

Splines on triangulations



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- Practical request: stable dimension & local construction

Stable dimension

A stable dimension only depends on degree, smoothness, topology

(# vertices, edges, triangles in $\mathcal{T}_0$)

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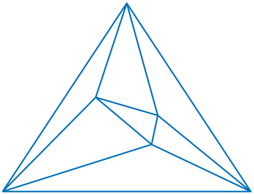
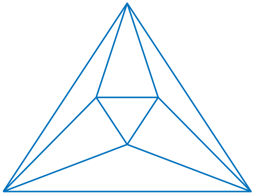


$\mathbb{S}_2^1(\mathcal{T}_0)$



dim $= 6$

dim $= 7$

Morgan–Scott triangulation

Stable dimension

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$$\mathbb{S}_p^r(\mathcal{T}_0) := \{f \in C^r(\Omega) : f_{|\Delta} \in \mathbb{P}_p, \ \forall \ \Delta \in \mathcal{T}_0\}$$

$$\text{Lower Bound} \leq \dim(\mathbb{S}_p^r(\mathcal{T}_0)) \leq \text{Upper Bound}$$

L. Schumaker, *Bounds on the dimension of spaces of multivariate piecewise polynomials*, Rocky Mnt. Math., 1984

A. Ibrahim, L. Schumaker, *Super spline spaces of smoothness r and degree $d \geq 3r+2$*, Constr. Approx., 1991

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$$\boxed{\dim(\mathbb{S}_3^1(\mathcal{T}_0)) = ???}$$

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Why smooth splines of low degrees?

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  Single element construction (finite element):

  Degree $4r+1$ for C^r: $\to \mathbb{S}_5^1$ (Argyris), $\mathbb{S}_9^2$, $\mathbb{S}_{13}^3$, $\ldots$

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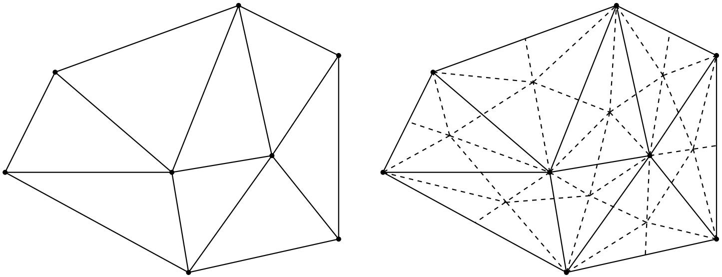
Solution: consider refined triangulations

Triangulation with subpatches

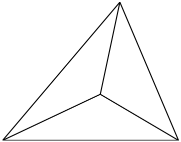


A splitting $\mathcal{T}_1$ of each triangle in $\mathcal{T}_0$ into subpatches:

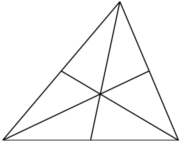
$$\mathbb{S}_p^r(\mathcal{T}_1) := \{f \in C^r(\Omega) : f_{|T} \in \mathbb{P}_p, \ \forall \ T \in \mathcal{T}_1\}$$

M.J. Lai and L.L. Schumaker, *Spline Functions on Triangulations*, Cambridge University Press, 2007

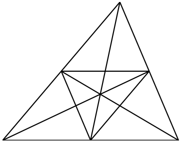
Triangulation with subpatches: popular splits



CT



PS6



PS12

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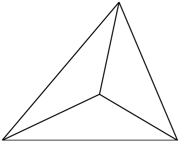
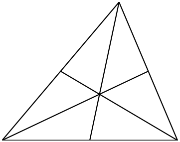
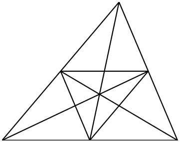


CT $\mathbb{S}^1_3$ $\mathbb{S}^2_7$ $\mathbb{S}^3_9$



PS6 $\mathbb{S}^1_2$ $\mathbb{S}^2_5$ $\mathbb{S}^3_7$



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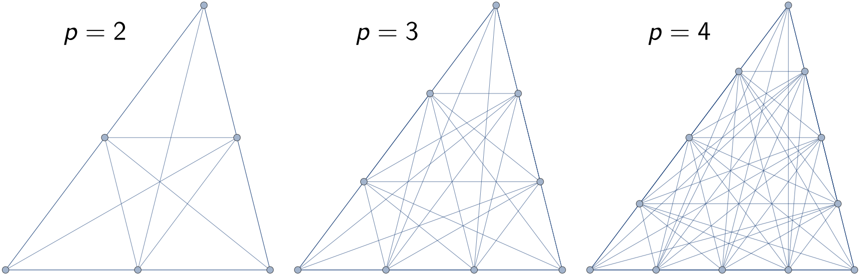
Triangulation with subpatches: Wang–Shi splits

WS_p-split: complete graph of $p+1$ uniform points on each edge



R.H. Wang and X.Q. Shi, $S^{\mu}_{\mu+1}$ *surface interpolations over triangulations*, in: Approximation, Optimization and Computing: Theory and Applications, Elsevier Science Publishers, pp. 205–208, 1990

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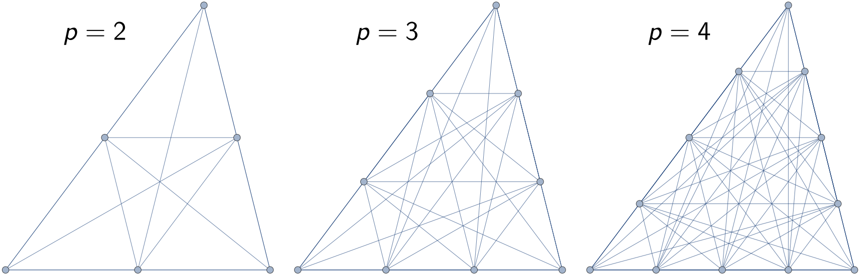

- $\mathbb{S}_p^{p-1}(\Delta_{\mathrm{WS}_p})$ on such partition Δ_{WS_p}
- cross-cut partitions

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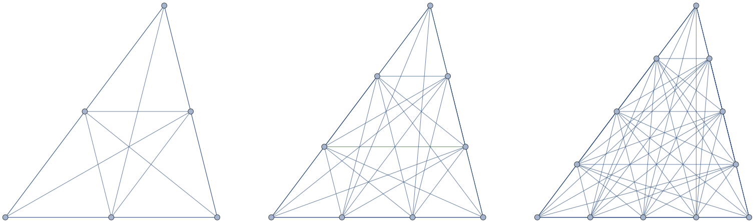
Dimension formula

On a cross-cut partition Δ_c of a simply connected domain in $\mathbb{R}^2$ with m interior cross cuts, we have $\dim \mathbb{S}_p^{p-1}(\Delta_c) = \dim \mathbb{P}_p + m, \; p \in \mathbb{N}$ provided at most $p+1$ lines cross at each interior vertex



WS_p-split: $m = 3p(p-1)$ cross cuts

T. Lyche, C. Manni, and H. Speleers, *Construction of C^2 cubic splines on arbitrary triangulations*, Found. Comput. Math., 22:1309–1350, 2022

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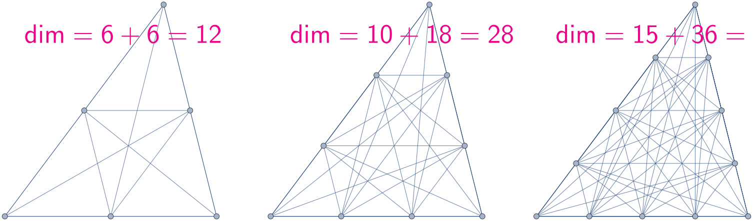

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Dimension formula: local and global

- Dimension comparisons with no split Δ:
  - $\dim(\mathbb{S}_2^1($$)) = 12 \quad \Leftrightarrow \quad \dim(\mathbb{S}_5^1(\Delta)) = 21$
  - $\dim(\mathbb{S}_3^2($$)) = 28 \quad \Leftrightarrow \quad \dim(\mathbb{S}_9^2(\Delta)) = 55$
  - $\dim(\mathbb{S}_4^3($$)) = 51 \quad \Leftrightarrow \quad \dim(\mathbb{S}_{13}^3(\Delta)) = 105$

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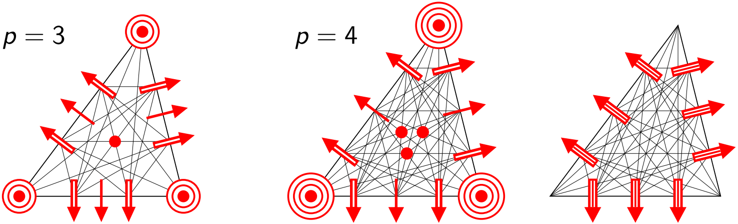
- Degrees of freedom for Δ_{WS_p}:



  - $\dim(\mathbb{S}^2_3(\mathcal{T}_1)) = \ \ 6n_v + 3n_e + n_t$
  - $\dim(\mathbb{S}^3_4(\mathcal{T}_1)) = 10n_v + 6n_e + 3n_t$

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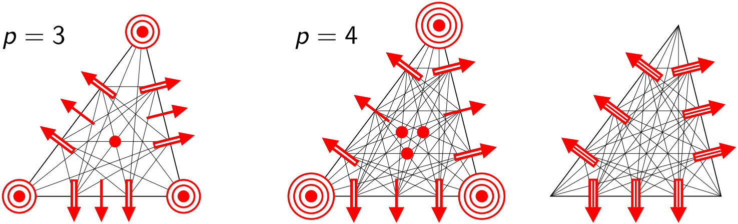
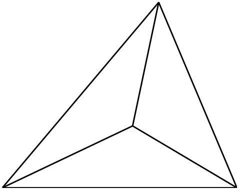


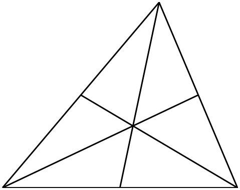
$$\dim(\mathbb{S}^{p-1}_p(\mathcal{T}_1)) = \binom{p+1}{2} n_v + \binom{p}{2} n_e + \binom{p-1}{2} n_t$$

Macro-triangles: many subpatches. . .

C^1 cubic



C^1 quadratic



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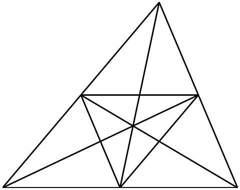

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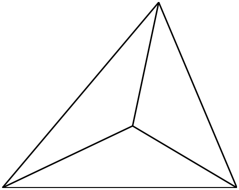

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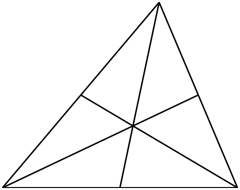

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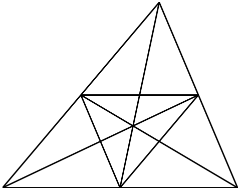
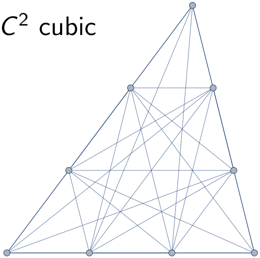
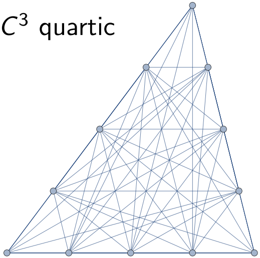


C^2 cubic



75 subpatches

C^3 quartic



250 subpatches

Looking for a good basis

The set of Bernstein polynomials are defined by

$$B_{i,j,k}(u,v,w) = \frac{p!}{i!j!k!} u^i v^j w^k, \quad i+j+k=p$$

where (u, v, w) are the barycentric coordinates w.r.t. triangle

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They form a basis for the polynomial space $\mathbb{P}_p$:

$$g \in \mathbb{P}_p \quad \Rightarrow \quad g(u,v,w) = \sum_{i+j+k=p} c_{i,j,k} B_{i,j,k}(u,v,w)$$

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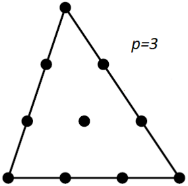
Looking for a good basis

Properties of Bernstein polynomials:

- nonnegative partition of unity
- differentiation formula, recurrence relation
- domain points and control points



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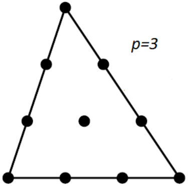
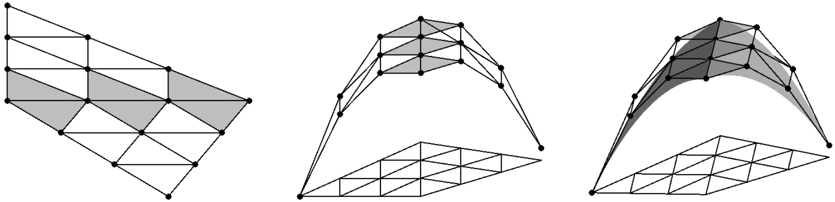


- simple conditions for smooth joins to neighboring triangles



Looking for a good basis

Building a global basis on a triangulation using the MDS principle:

A minimal determining set (MDS) is a set with the minimal number of domain points such that the corresponding coefficients equal to zero give the zero function

- determine all smoothness conditions
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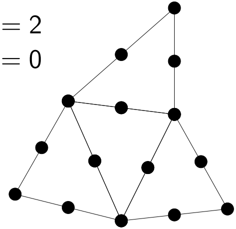
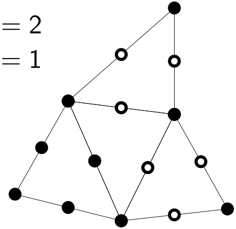
$r = 0$



$p = 2$
$r = 1$



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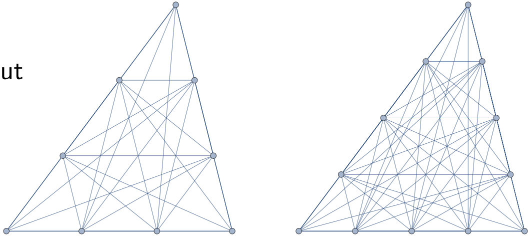
But... what about



?

Looking for a very good basis

We would like a B-spline like basis for $\mathbb{S}_p^{p-1}(\Delta_{\mathrm{WS}_p})$:

- local support
- nonnegative partition of unity
- differentiation formula, recurrence relation
- Marsden-like identity: explicit representation of polynomials

  $\Rightarrow$ can be used to build dual functionals and quasi-interpolants
- simple conditions for smooth joins to neighboring triangles

  $\Rightarrow$ can be used to build a global MDS basis

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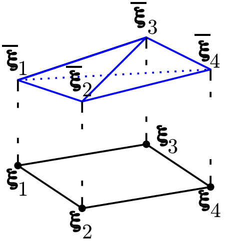
Try simplex splines (multivariate B-splines)

C.A. Micchelli, *On a numerically efficient method for computing multivariate B-splines*, in: Multivariate Approximation Theory, Birkhäuser, pp. 211–248, 1979

W. Dahmen, *On multivariate B-splines*, SIAM J. Numer. Anal., 17:179–191, 1980

Simplex lifting



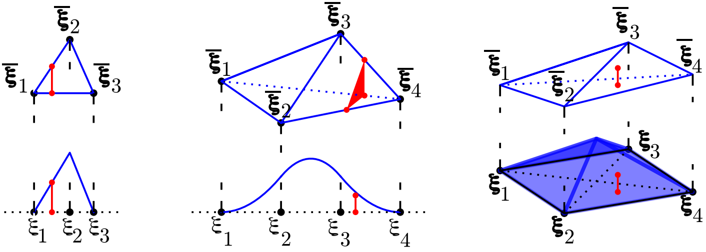
$s = 2,\ n = 3$

- Given a sequence of $n+1$ points $\Xi := \{\boldsymbol{\xi}_1, \ldots, \boldsymbol{\xi}_{n+1}\}$ in $\mathbb{R}^s$
- Assume nondegenerate $\langle \Xi \rangle$: $\mathrm{vol}_s(\langle \Xi \rangle) > 0$
- $\overline{\boldsymbol{\xi}}_1, \ldots, \overline{\boldsymbol{\xi}}_{n+1}$ points in $\mathbb{R}^n$ whose projection $\pi : \mathbb{R}^n \to \mathbb{R}^s$ onto the first s coordinates satisfies $\pi(\overline{\boldsymbol{\xi}}_i) = \boldsymbol{\xi}_i$ for $i = 1, \ldots, n+1$.
- The simplex $\sigma := \langle \overline{\boldsymbol{\xi}}_1, \ldots, \overline{\boldsymbol{\xi}}_{n+1} \rangle$ has positive volume

Simplex splines: geometric definition

- $\Xi := \{\boldsymbol{\xi}_1, \ldots, \boldsymbol{\xi}_{n+1}\} \in \mathbb{R}^s$ (knots)
- $\sigma := \langle \overline{\boldsymbol{\xi}}_1, \ldots \overline{\boldsymbol{\xi}}_{n+1} \rangle \in \mathbb{R}^n$ (lifted simplex)
- $p := n - s \geq 0$ (degree)

$$M_\Xi : \mathbb{R}^s \to \mathbb{R}, \quad M_\Xi(\boldsymbol{x}) := \begin{cases} \dfrac{\mathrm{vol}_p(\sigma \cap \pi^{-1}(\boldsymbol{x}))}{\mathrm{vol}_n(\sigma)}, & \text{if } \mathrm{vol}_s(\langle \Xi \rangle) > 0 \\ 0, & \text{otherwise} \end{cases}$$



univariate linear univariate quadratic bivariate linear

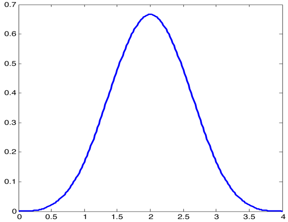
Simplex splines: properties

- Knot dependence: M_Ξ only depends on Ξ; in particular, it is independent of the choice of σ and the ordering of the knots
- Nonnegativity: M_Ξ is a nonnegative piecewise polynomial of total degree p and support $\langle \Xi \rangle$
- Normalization: M_Ξ has unit integral
- The ABC recurrence relations ($n = p + s$):
  - *Differentiation formula (A-recurrence)*: For any $\boldsymbol{u} \in \mathbb{R}^s$
    $$(\boldsymbol{u} \cdot \nabla) M_\Xi = (p+s) \sum_{i=1}^{p+s+1} a_i M_{\Xi \setminus \boldsymbol{\xi}_i}, \quad \sum_i a_i \boldsymbol{\xi}_i = \boldsymbol{u}, \ \sum_i a_i = 0$$
  - *Recurrence relation (B-recurrence)*: For any $\boldsymbol{x} \in \mathbb{R}^s$
    $$M_\Xi(\boldsymbol{x}) = \frac{p+s}{p} \sum_{i=1}^{p+s+1} b_i M_{\Xi \setminus \boldsymbol{\xi}_i}(\boldsymbol{x}), \quad \sum_i b_i \boldsymbol{\xi}_i = \boldsymbol{x}, \ \sum_i b_i = 1$$
  - *Knot insertion formula (C-recurrence)*: For any $\boldsymbol{y} \in \mathbb{R}^s$
    $$M_\Xi = \sum_{i=1}^{p+s+1} c_i M_{\Xi \cup \boldsymbol{y} \setminus \boldsymbol{\xi}_i}, \quad \sum_i c_i \boldsymbol{\xi}_i = \boldsymbol{y}, \ \sum_i c_i = 1$$

Simplex splines: univariate case

$$s = 1$$

- M_Ξ = univariate B-spline of degree p with knots Ξ
  normalized to have integral one in the nondegenerate case



Simplex splines: bivariate case

$$s = 2$$

The lines in the complete graph of Ξ are called knot lines, providing a partition of $\langle \Xi \rangle$ into polygonal regions

- The simplex spline M_Ξ is a polynomial of degree

$$p = \#\Xi - 3$$

  in each region of this partition

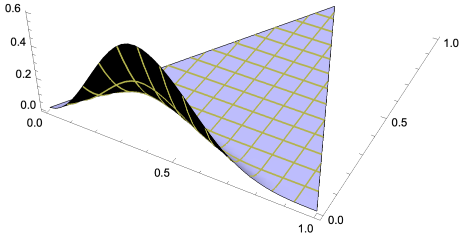
- Across a knot line:

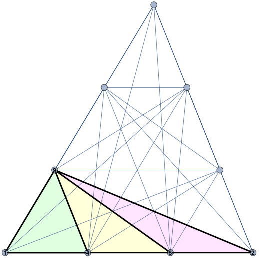
$$M_\Xi \in C^{p+1-\mu}$$

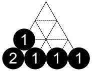
  $\mu =$ number of knots on that knot line, including multiplicities

- At any boundary line: zero or univariate B-spline

Simplex splines: cubic example, $p = 3$, $s = 2$

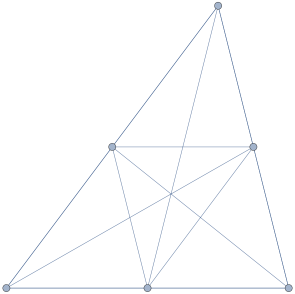






- $p + s + 1 = 6$ knots with double knot at vertex
- Across horizontal boundary line: $C^{p+1-\mu} = C^{4-5} = C^{-1}$
- Univariate cubic B-spline restricted to that line
- $C^{p+1-\mu} = C^{4-3} = C^1$ across left boundary line
- Across remaining knot lines: $C^{p+1-\mu} = C^{4-2} = C^2$

C^1 quadratics

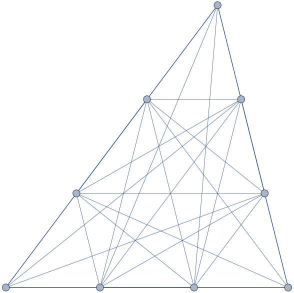


A nonnegative partition of unity (scaled) simplex spline basis

$$B_1, \ldots, B_{12}$$

E. Cohen, T. Lyche, and R.F. Riesenfeld, *A B-spline-like basis for the Powell–Sabin 12-split based on simplex splines*, Math. Comput., 82:1667–1707, 2013

C^2 cubics

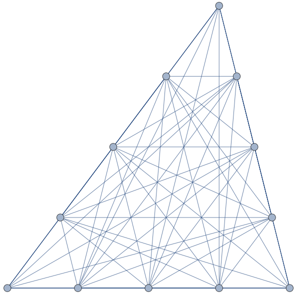


A nonnegative partition of unity (scaled) simplex spline basis

$$B_1, \dots, B_{28}$$

T. Lyche, C. Manni, and H. Speleers, *Construction of C^2 cubic splines on arbitrary triangulations*, Found. Comput. Math., 22:1309–1350, 2022

C^3 quartics

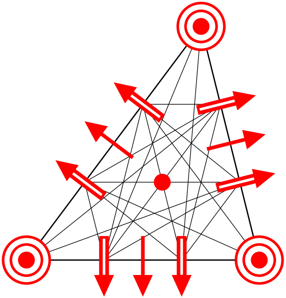


A nonnegative partition of unity (scaled) simplex spline basis

$$B_1, \ldots, B_{51}$$

T. Lyche, C. Manni, and H. Speleers, *A local simplex spline basis for C^3 quartic splines on arbitrary triangulations*, Appl. Math. Comput., 462:128330, 2024

C^2 cubics: details of construction



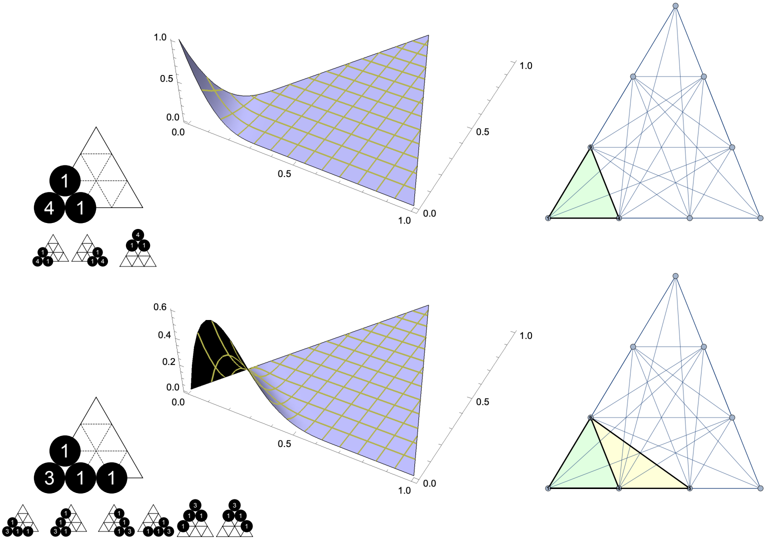
A nonnegative partition of unity (scaled) simplex spline basis

$$B_1, \ldots, B_{28}$$

T. Lyche, C. Manni, and H. Speleers, *Construction of C^2 cubic splines on arbitrary triangulations*, Found. Comput. Math., 22:1309–1350, 2022

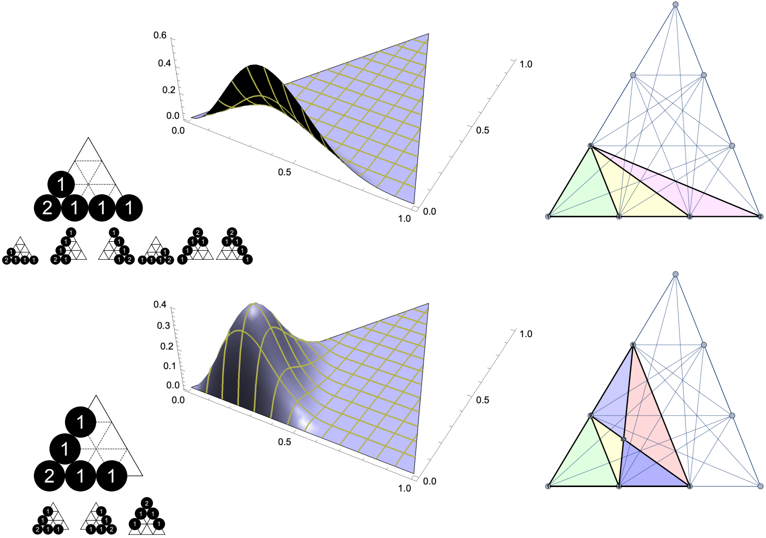
C^2 cubics: construction of $B_1, \ldots, B_9$

$B_1, \ldots, B_9$ related to value and first derivatives at vertices



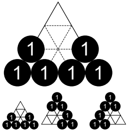
C^2 cubics: construction of $B_{10}, \ldots, B_{18}$

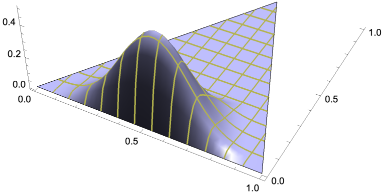
$B_{10}, \ldots, B_{18}$ related to second derivatives at vertices

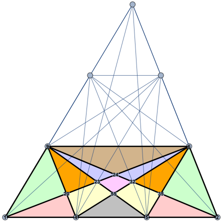


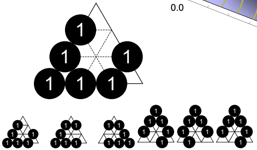
C^2 cubics: construction of $B_{19}, \ldots, B_{27}$

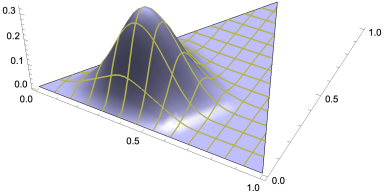
$B_{19}, \ldots, B_{27}$ related to first and second derivatives across edges

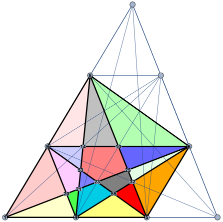






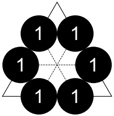


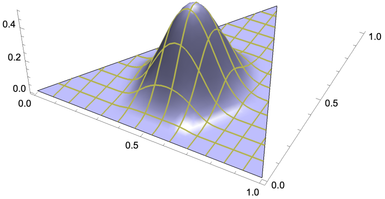


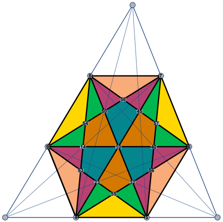


C^2 cubics: construction of B_{28}

B_{28} related to interior value







C^{p-1} splines of degree p: properties

Properties of the (scaled) simplex spline basis $p = 2, 3, 4$:

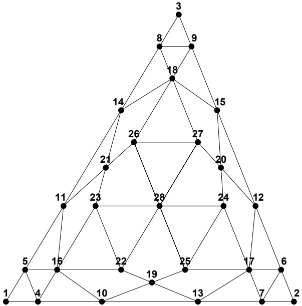
- Basis: they are linearly independent on $\Delta \Rightarrow$ basis of $\mathbb{S}_p^{p-1}(\Delta_{\mathrm{WS}_p})$
- Nonnegativity: they form a nonnegative partition of unity
- Classical B-splines when restricted to boundary
- Simplex recurrence relations for their manipulation
- Marsden-like identity: representation of polynomials

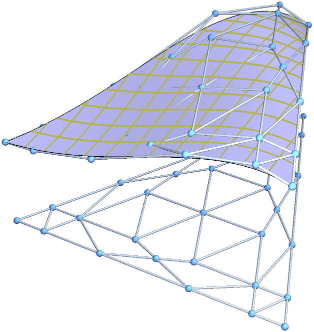
$$(1 + \boldsymbol{y}^T \boldsymbol{x})^p = \sum_i \psi_i(\boldsymbol{y}) B_i(\boldsymbol{x}), \quad \boldsymbol{y} \in \mathbb{R}^2, \quad \boldsymbol{x} \in \Delta$$

  with $\psi_i(\boldsymbol{y}) := \prod_{j=1}^{p} (1 + \boldsymbol{y}^T \boldsymbol{r}_{i,j})$ for almost all i

- They can be extended in a Bernstein–Bézier fashion to compute smooth surfaces on arbitrary triangulations

Domain points and control net for $p = 3$

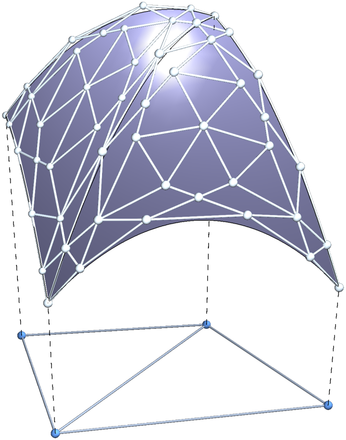




Domain points and a possible control net for $p = 3$

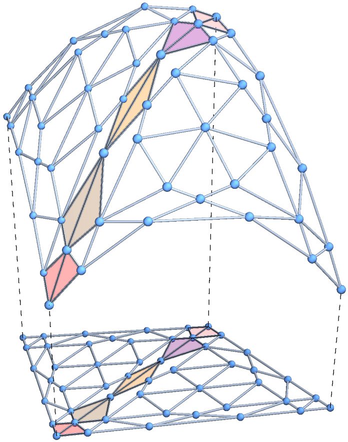
The control mesh is at a distance $O(h^2)$ from the surface where h is the longest side of the triangle

Smoothness across an edge for $p = 3$



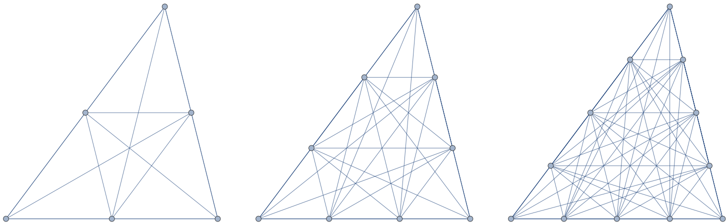
C^0, C^1, C^2 smoothness conditions analogous to the Bernstein representation in triangular polynomial case

Smoothness across an edge for $p = 3$



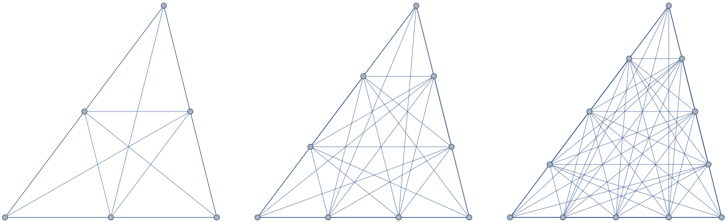
C^1 conditions in the C^2 cubic case

Conclusions



- The WS_p-splits, $p = 2, 3, 4$ allow to locally construct $C^1/C^2/C^3$ quadratic/cubic/quartic splines on any triangulation
- They seem extremely complicated, but
  - the computation can be done in a Bernstein–Bézier fashion using a simplex spline basis on each macro-triangle which forms a nonnegative partition of unity
  - for their manipulation one can exploit the features of simplex splines (recurrence relations)

Conclusions



- The WS_p-splits, $p = 2, 3, 4$ allow to locally construct $C^1/C^2/C^3$ quadratic/cubic/quartic splines on any triangulation
- Local approximation methods can be developed by exploiting Marsden-like identity
- Tailored quadrature rules can be constructed by exploiting the inter-element maximal smoothness
    - 4 nodes suffice for integration of C^1 quadratics
    - 10 nodes suffice for integration of C^2 cubics

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E. Cohen, T. Lyche, and R.F. Riesenfeld, *A B-spline-like basis for the Powell–Sabin 12-split based on simplex splines*, Math. Comput., 82:1667–1707, 2013

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Announcement

SIAM Conference on Computational Geometric Design

July 28–30, 2025
Montréal Convention Center
Montréal, Québec, Canada

https://www.siam.org/conferences-events/siam-conferences/gd25/

Keynote speakers
Henry Bucklow (ITI CADfix), Géraldine Morin (U Toulouse), Helmut Pottman (TU Wien), Alla Sheffer (U British Columbia), Wenping Wang (Texas A&M)

Deadline dates
Minisymposium proposal: January 13, 2025
Abstract submission: January 27, 2025