# MINKOWSKI PENALTIES

## Differentiable Geometric Constraint Enforcement

**Jiří Minarčík**, Sam Estep, Wode Ni, Keenan Crane

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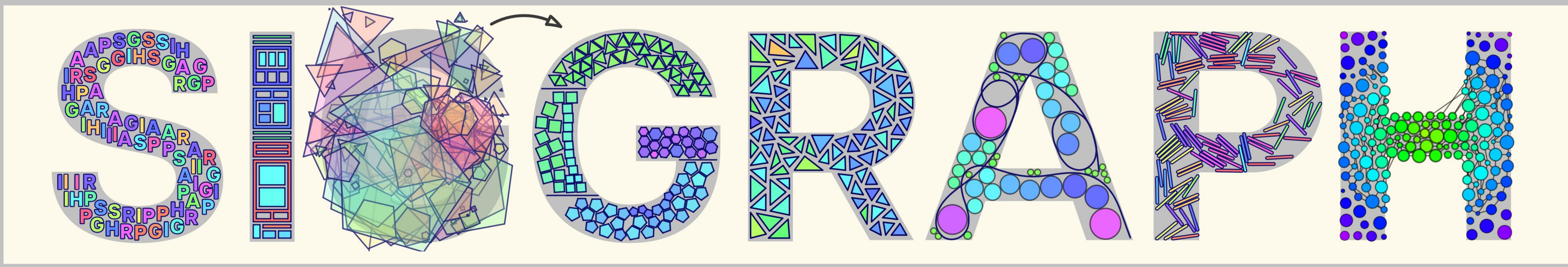
project page




### How to optimize two shapes so that they don't overlap?

We use signed distance function (SDF)

$$\phi_C(\mathbf{x}) := \begin{cases} -d(\mathbf{x}, \partial C) & \mathbf{x} \in C, \\ d(\mathbf{x}, \partial C) & \mathbf{x} \notin C. \end{cases}$$

of Minkowski difference $C = B - A$

$$B - A = \{\mathbf{a} - \mathbf{b} \colon \mathbf{a} \in A, \mathbf{b} \in B\}.$$

Given a set of Minkowski penalties $\mathcal{P}_i$ and additional energy functionals $\mathcal{E}_i$ we aim to solve the following problem:

$$\min_{\mathbf{p} \in \mathbb{R}^m} \sum_{i=1}^{k} \mathcal{E}_i(\mathbf{p}) \quad \text{s.t.} \quad \sum_{i=1}^{l} \mathcal{P}_i(\mathbf{p}) = 0.$$

OVERLAPPING

$$\mathcal{P}_o(A, B) = \max(0, \phi_C(\mathbf{0}))$$

NON-OVERLAPPING

$$\mathcal{P}_d(A, B) = -\min(0, \phi_C(\mathbf{0}))$$

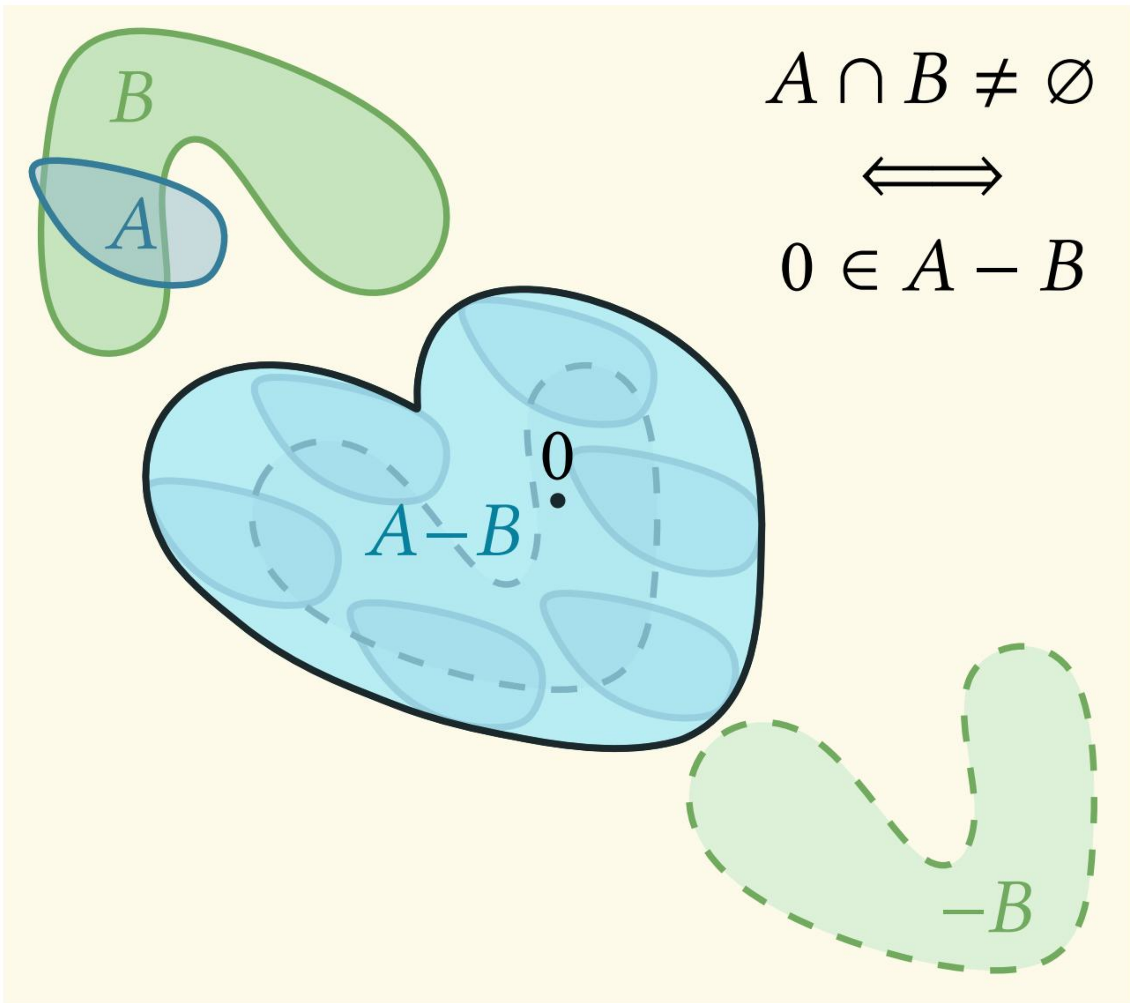
TANGENT

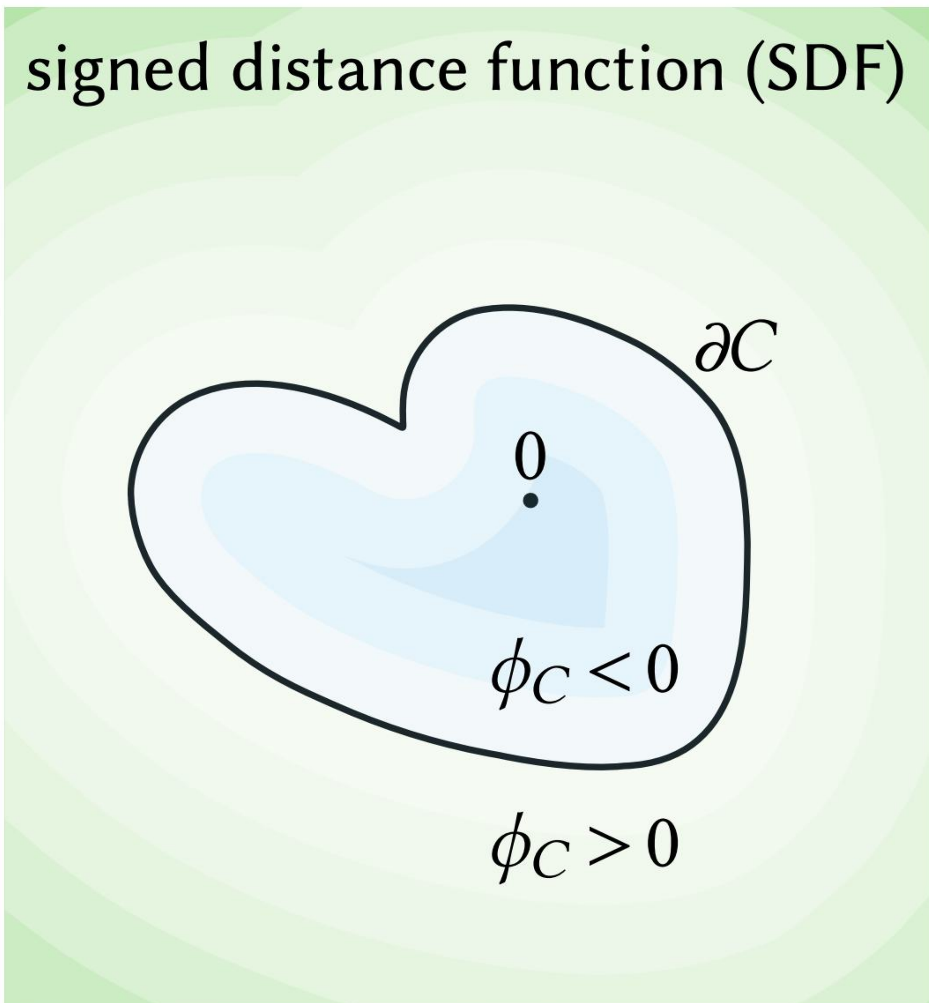
$$\mathcal{P}_t(A, B) = |\phi_C(\mathbf{0})|$$

NESTED

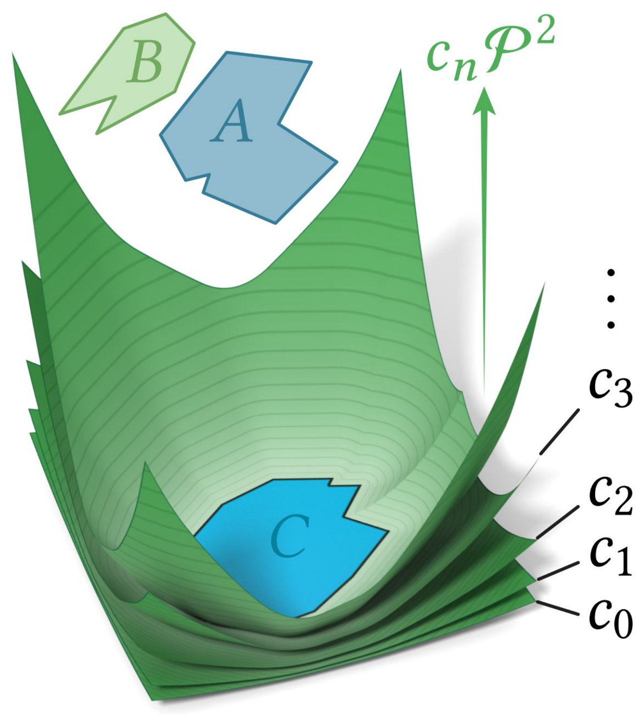
$$\mathcal{P}_c(A, B) = \mathcal{P}_d(A, B^c)$$

...




### How to minimize this energy?

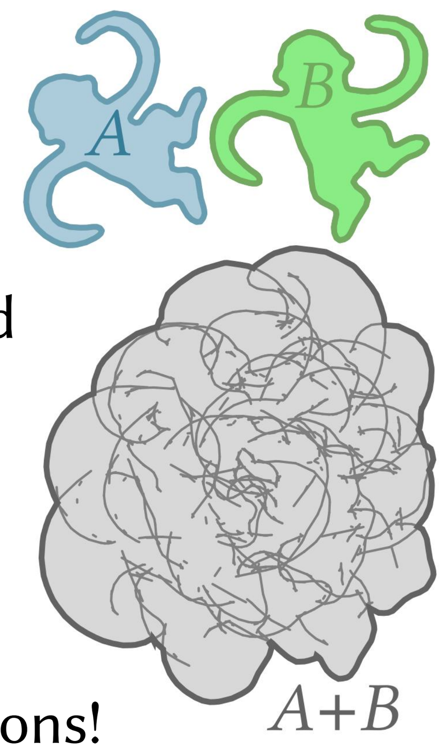



Exterior point method:

$$\min_{\mathbf{p}} \sum_{i=1}^{k} \mathcal{E}_i + c_n \sum_{i=1}^{l} \mathcal{P}_i^2$$

Individual problems solved using L-BFGS with step size given by a line search strategy.

Gradients are computed using autodif engine Rose. Shapes are approximated by polygons and their Minkowski sum is computed using reduced convolution. For rigit translations, the sum can be precomputed.




Works even for non-convex polygons!

### Examples and Applications